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The electromagnetic optical theorem revisited

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Abstract

We revisit the optical theorem relevant to the far-field electromagnetic scattering by an arbitrary particle. We compute both the Poynting vector and the energy density of the total field and demonstrate once again that, despite a recent claim to the contrary, the extinction is caused by the interference of the incident and the forward-scattered field. However, caution must be exercised when one describes electromagnetic scattering using a formalism based on the coherency dyad since this approach may lead to unphysical artifacts.
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1. Introduction

The electromagnetic optical theorem is a well-established fact which states that extinction is caused by the interference of the incident and the forward-scattered field [1,2]. However, a recent publication [3] emphasized backward scattering within the framework of the optical theorem and thereby revived interest in this topic. Although it has already been argued that the conclusion drawn in [3] is wrong [4,5], we feel that it would be worthwhile to revisit the optical theorem by computing the energy density of the total field in addition to its Poynting vector. Another objective of this short note is to analyze what happens when one attempts to describe electromagnetic scattering using a formalism based on the coherency dyad. In order to avoid redundancy and save space, we will assume that the reader is familiar with our recent book on electromagnetic scattering [6] and will use exactly the same terminology and notation.

2. Electromagnetic power

Consider scattering of a plane electromagnetic wave by an arbitrary particle imbedded in an infinite, homogeneous, linear, isotropic, and nonabsorbing medium. The particle and the host medium are assumed to be nonmagnetic. The standard measurement configuration involves a well-collimated detector of electromagnetic radiation located at a distance r from the scattering object in the far-field zone, with its sensitive surface aligned normal to and centered on the position vector $\mathbf{r} = r\hat{\mathbf{r}}$ (see Fig. 1). The origin of the

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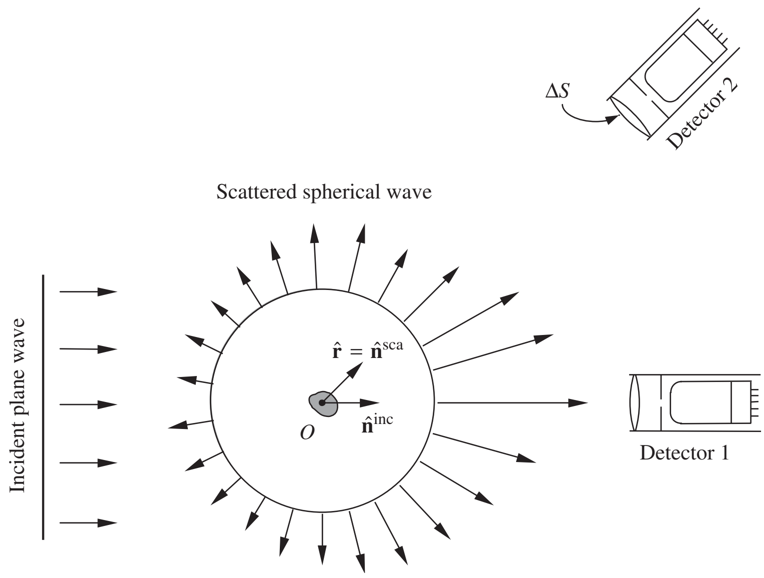

Fig. 1. Standard measurement configuration.

laboratory reference frame is inside the scattering object, the incidence direction is specified by the unit vector $\hat{\mathbf{n}}^{\text{inc}}$, and the scattering direction is given by the unit vector $\hat{\mathbf{n}}^{\text{sca}} = \hat{\mathbf{r}}$.

The functional definition of a well-collimated detector suitable for our purposes is that of a sensitive plane surface of an area ΔS that registers the energy of monochromatic light impinging on any point of ΔS in directions confined to a small solid angle Ω (called the detector angular aperture) centered at the local normal to the detector surface. We assume that the diameter D of the detector surface is significantly greater than any linear dimension a of the scattering object:

$$D \gg a. \tag{1}$$

This will ensure that the right-hand side of Eq. (16) below is positive, thereby making possible a meaningful measurement of extinction. We will also assume that the angular size of the detector as viewed from the scatterer is small so that the scattered light impinging on different parts of the detector surface propagates in approximately the same direction. This is equivalent to requiring that

$$r \gg \frac{D}{2}. \tag{2}$$

Furthermore, we assume that the angular size of the detector sensitive surface as seen from the scattering object is smaller than the detector angular aperture:

$$\frac{\Delta S}{r^2} < \Omega. \tag{3}$$

This ensures that all radiation scattered by the object in radial directions and impinging on ΔS is detected.

The total field at an external point can be represented by the vector sum of the incident and scattered fields:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^{\text{inc}}(\mathbf{r}) + \mathbf{E}^{\text{sca}}(\mathbf{r}), \tag{4}$$

$$\mathbf{H}(\mathbf{r}) = \mathbf{H}^{\text{inc}}(\mathbf{r}) + \mathbf{H}^{\text{sca}}(\mathbf{r}), \tag{5}$$

where the common time-harmonic factor $\exp(-\mathrm{i}\omega t)$ (ω is the angular frequency and t is time) is omitted for the sake of brevity. We begin by writing the time-averaged Poynting vector $\langle \mathbf{S}(\mathbf{r}', t)\rangle_t$ at any point on the sensitive surface of the detector as the sum of three terms:

$$\begin{aligned}\langle \mathbf{S}(\mathbf{r}', t)\rangle_t &= \tfrac{1}{2}\mathrm{Re}[\mathbf{E}(\mathbf{r}') \times \mathbf{H}^*(\mathbf{r}')] \\ &= \langle \mathbf{S}^{\mathrm{inc}}(\mathbf{r}', t)\rangle_t + \langle \mathbf{S}^{\mathrm{sca}}(\mathbf{r}', t)\rangle_t + \langle \mathbf{S}^{\mathrm{ext}}(\mathbf{r}', t)\rangle_t,\end{aligned} \tag{6}$$

where an asterisk denotes complex conjugation, $\mathbf{r}' = r'\hat{\mathbf{r}}'$ is the corresponding radius vector connecting the particle and the observation point,

$$\langle \mathbf{S}^{\mathrm{inc}}(\mathbf{r}', t)\rangle_t = \tfrac{1}{2}\mathrm{Re}[\mathbf{E}^{\mathrm{inc}}(\mathbf{r}') \times \mathbf{H}^{\mathrm{inc}*}(\mathbf{r}')] \tag{7}$$

and

$$\langle \mathbf{S}^{\mathrm{sca}}(\mathbf{r}', t)\rangle_t = \tfrac{1}{2}\mathrm{Re}[\mathbf{E}^{\mathrm{sca}}(\mathbf{r}') \times \mathbf{H}^{\mathrm{sca}*}(\mathbf{r}')] \tag{8}$$

are the Poynting vectors associated with the incident and the scattered field, respectively, and

$$\langle \mathbf{S}^{\mathrm{ext}}(\mathbf{r}', t)\rangle_t = \tfrac{1}{2}\mathrm{Re}[\mathbf{E}^{\mathrm{inc}}(\mathbf{r}') \times \mathbf{H}^{\mathrm{sca}*}(\mathbf{r}') + \mathbf{E}^{\mathrm{sca}}(\mathbf{r}') \times \mathbf{H}^{\mathrm{inc}*}(\mathbf{r}')] \tag{9}$$

can be interpreted as the term caused by interaction between the incident and the scattered field.

Recalling Eq. (A.8) of [6], we have for the incident plane wave in the far-field zone of the scattering particle:

$$\begin{aligned}\mathbf{E}^{\mathrm{inc}}(\mathbf{r}') &= \mathbf{E}_0^{\mathrm{inc}} \exp(\mathrm{i}k_1 \hat{\mathbf{n}}^{\mathrm{inc}} \cdot \mathbf{r}') \\ &\underset{k_1 r' \to \infty}{=} \frac{\mathrm{i}2\pi}{k_1}\left[\delta(\hat{\mathbf{n}}^{\mathrm{inc}} + \hat{\mathbf{r}}')\frac{\exp(-\mathrm{i}k_1 r')}{r'} - \delta(\hat{\mathbf{n}}^{\mathrm{nic}} - \hat{\mathbf{r}}')\frac{\exp(\mathrm{i}k_1 r')}{r'}\right]\mathbf{E}_0^{\mathrm{inc}},\end{aligned} \tag{10}$$

$$\mathbf{E}_0^{\mathrm{inc}} \cdot \hat{\mathbf{n}}^{\mathrm{inc}} = 0, \tag{11}$$

$$\begin{aligned}\mathbf{H}^{\mathrm{inc}}(\mathbf{r}') &= \sqrt{\frac{\varepsilon_1}{\mu_0}} \exp(\mathrm{i}k_1 \hat{\mathbf{n}}^{\mathrm{inc}} \cdot \mathbf{r}')\hat{\mathbf{n}}^{\mathrm{inc}} \times \mathbf{E}_0^{\mathrm{inc}} \\ &\underset{k_1 r' \to \infty}{=} \frac{\mathrm{i}2\pi}{k_1}\left[\delta(\hat{\mathbf{n}}^{\mathrm{inc}} + \hat{\mathbf{r}}')\frac{\exp(-\mathrm{i}k_1 r')}{r'} - \delta(\hat{\mathbf{n}}^{\mathrm{inc}} - \hat{\mathbf{r}}')\frac{\exp(\mathrm{i}k_1 r')}{r'}\right]\sqrt{\frac{\varepsilon_1}{\mu_0}}\hat{\mathbf{n}}^{\mathrm{inc}} \times \mathbf{E}_0^{\mathrm{inc}},\end{aligned} \tag{12}$$

where $k_1 = \omega(\varepsilon_1\mu_0)^{1/2}$ is the wave number in the host medium, ε_1 is the electric permittivity of the host medium, μ_0 is the magnetic permeability of free space, and $\delta(\hat{\mathbf{r}})$ is the solid-angle delta function. For the scattered spherical wave, we have

$$\mathbf{E}^{\mathrm{sca}}(\mathbf{r}') \underset{k_1 r' \to \infty}{=} \frac{\exp(\mathrm{i}k_1 r')}{r'}\mathbf{E}_1^{\mathrm{sca}}(\hat{\mathbf{r}}'), \quad \mathbf{E}_1^{\mathrm{sca}}(\hat{\mathbf{r}}') \cdot \hat{\mathbf{r}}' = 0, \tag{13}$$

$$\begin{aligned}\mathbf{H}^{\mathrm{sca}}(\mathbf{r}') &= \frac{1}{\mathrm{i}\omega\mu_0}\nabla \times \mathbf{E}^{\mathrm{sca}}(\mathbf{r}') \\ &\underset{k_1 r' \to \infty}{=} \sqrt{\frac{\varepsilon_1}{\mu_0}}\frac{\exp(\mathrm{i}k_1 r')}{r'}\hat{\mathbf{r}}' \times \mathbf{E}_1^{\mathrm{sca}}(\hat{\mathbf{r}}').\end{aligned} \tag{14}$$

One can now derive that the total electromagnetic power received by a well-collimated detector is

$$\begin{aligned}W_{\Delta S}(\hat{\mathbf{r}}) &= \int_{\Delta S} \mathrm{d}S\hat{\mathbf{r}}' \cdot \langle \mathbf{S}(\mathbf{r}', t)\rangle_t \\ &\approx \frac{1}{2}\sqrt{\frac{\varepsilon_1}{\mu_0}}\frac{\Delta S}{r^2}|\mathbf{E}_1^{\mathrm{sca}}(\hat{\mathbf{r}})|^2\end{aligned} \tag{15}$$

when $\hat{\mathbf{r}} \neq \hat{\mathbf{n}}^{\mathrm{inc}}$ (detector 2 in Fig. 1), whereas for the exact forward-scattering direction (detector 1),

$$\begin{aligned} W_{\Delta S}(\hat{\mathbf{n}}^{\mathrm{inc}}) &= \int_{\Delta S} \mathrm{d}S \hat{\mathbf{r}}' \cdot \langle \mathbf{S}(\mathbf{r}', t) \rangle_t \\ &\approx \frac{1}{2} \Delta S \sqrt{\frac{\varepsilon_1}{\mu_0}} \left[|\mathbf{E}_0^{\mathrm{inc}}|^2 + \frac{1}{r^2} |\mathbf{E}_1^{\mathrm{sca}}(\hat{\mathbf{n}}^{\mathrm{inc}})|^2 \right] + r^2 \int_{\Delta\Omega} \mathrm{d}\hat{\mathbf{r}}' \hat{\mathbf{n}}^{\mathrm{inc}} \cdot \langle \mathbf{S}^{\mathrm{ext}}(r\hat{\mathbf{r}}', t) \rangle_t \\ &\approx \frac{1}{2} \Delta S \sqrt{\frac{\varepsilon_1}{\mu_0}} \left[|\mathbf{E}_0^{\mathrm{inc}}|^2 + \frac{1}{r^2} |\mathbf{E}_1^{\mathrm{sca}}(\hat{\mathbf{n}}^{\mathrm{inc}})|^2 \right] - \frac{2\pi}{k_1} \sqrt{\frac{\varepsilon_1}{\mu_0}} \mathrm{Im}[\mathbf{E}_1^{\mathrm{sca}}(\hat{\mathbf{n}}^{\mathrm{inc}}) \cdot \mathbf{E}_0^{\mathrm{inc}*}] \\ &= \frac{1}{2} \sqrt{\frac{\varepsilon_1}{\mu_0}} |\mathbf{E}_0^{\mathrm{inc}}|^2 \Delta S - \frac{2\pi}{k_1} \sqrt{\frac{\varepsilon_1}{\mu_0}} \mathrm{Im}[\mathbf{E}_1^{\mathrm{sca}}(\hat{\mathbf{n}}^{\mathrm{inc}}) \cdot \mathbf{E}_0^{\mathrm{inc}*}] + O(r^{-2}), \end{aligned} \tag{16}$$

where $\Delta\Omega = \Delta S / r^2$ is the solid angle centered at the direction $\hat{\mathbf{n}}^{\mathrm{inc}}$ and formed by the detector surface at the distance r from the particle.

The first term on the right-hand side of Eq. (16) is proportional to the detector area ΔS and is equal to the electromagnetic power that would be received by detector 1 in the absence of the scattering object, whereas the second term is independent of ΔS and describes attenuation caused by interposing the object between the light source and the detector. Thus, the detector centered at the exact forward-scattering direction measures the power of the incident light attenuated by the interference of the incident and the scattered field plus a relatively small contribution from the scattered light, whereas the detector centered at any other direction registers only the scattered light.

Eq. (16) represents a well-known form of the optical theorem [6] and demonstrates that the signal measured by the detector facing the incident plane wave is attenuated owing to the interference of the incident wave and the spherical wave scattered in the exact *forward* direction. With regard to the claim made in Ref. [3], it is interesting to note that the presence of the terms proportional to the delta function $\delta(\hat{\mathbf{n}}^{\mathrm{inc}} + \hat{\mathbf{r}})$ on the right-hand sides of Eqs. (10) and (12) seems to indicate that there must be interference of the incident field and the field scattered in the exact backscattering direction. It is easy to verify, however, that the contribution of the interference term $\langle \mathbf{S}^{\mathrm{ext}}(\mathbf{r}', t) \rangle_t$ to the signal measured by a detector facing the exact backscattering direction vanishes upon taking the real part of the signal according to Eq. (9).

The physical interpretation of these results is rather transparent. In the forward direction, the scattered and incident waves are propagating in the same direction and we have traveling waves: this is what the Im part leaves. However, in the backward direction, the incident and scattered waves are traveling in opposite directions, leaving standing waves that cannot and do not contribute to the extinction.

3. Electromagnetic energy density

The optical theorem is usually formulated in terms of the Poynting vector. It is interesting, however, to supplement the above analysis by considering the spatial distribution of the electromagnetic energy density. The general formula for the time-averaged electromagnetic energy density of the total field at a point $\mathbf{r}$ is as follows:

$$\langle U(\mathbf{r}, t) \rangle_t = \tfrac{1}{4} [\varepsilon_1 \mathbf{E}(\mathbf{r}) \cdot \mathbf{E}^*(\mathbf{r}) + \mu_0 \mathbf{H}(\mathbf{r}) \cdot \mathbf{H}^*(\mathbf{r})]. \tag{17}$$

We thus have

$$\langle U(\mathbf{r}, t) \rangle_t = \langle U^{\mathrm{inc}}(\mathbf{r}, t) \rangle_t + \langle U^{\mathrm{sca}}(\mathbf{r}, t) \rangle_t + \langle U^{\mathrm{ext}}(\mathbf{r}, t) \rangle_t, \tag{18}$$

where

$$\begin{aligned} \langle U^{\mathrm{inc}}(\mathbf{r}, t) \rangle_t &= \tfrac{1}{4} [\varepsilon_1 \mathbf{E}^{\mathrm{inc}}(\mathbf{r}) \cdot \mathbf{E}^{\mathrm{inc}*}(\mathbf{r}) + \mu_0 \mathbf{H}^{\mathrm{inc}}(\mathbf{r}) \cdot \mathbf{H}^{\mathrm{inc}*}(\mathbf{r})] \\ &= \tfrac{1}{2} \varepsilon_1 |\mathbf{E}_0^{\mathrm{inc}}|^2 \end{aligned} \tag{19}$$

is the component due to the incident field,

$$\begin{aligned} \langle U^{\mathrm{sca}}(r, t) \rangle_t &= \tfrac{1}{4} [\varepsilon_1 \mathbf{E}^{\mathrm{sca}}(\mathbf{r}) \cdot \mathbf{E}^{\mathrm{sca}*}(\mathbf{r}) + \mu_0 \mathbf{H}^{\mathrm{sca}}(\mathbf{r}) \cdot \mathbf{H}^{\mathrm{sca}*}(\mathbf{r})] \\ &= \frac{\varepsilon_1 |\mathbf{E}_1^{\mathrm{sca}}(\hat{\mathbf{r}})|^2}{2r^2} \end{aligned} \tag{20}$$

is that due to the scattered field, and

$$\langle U^{\rm ext}(\mathbf{r},t)\rangle_t = \tfrac{1}{2}\,{\rm Re}[\varepsilon_1 \mathbf{E}^{\rm inc}(\mathbf{r})\cdot\mathbf{E}^{\rm sca*}(\mathbf{r}) + \mu_0 \mathbf{H}^{\rm inc}(\mathbf{r})\cdot\mathbf{H}^{\rm sca*}(\mathbf{r})]$$
$$= -\frac{2\pi\varepsilon_1}{k_1 r^2}\,\delta(\hat{\mathbf{n}}^{\rm inc}-\hat{\mathbf{r}})\,{\rm Im}[\mathbf{E}_1^{\rm sca}(\hat{\mathbf{n}}^{\rm inc})\cdot\mathbf{E}_0^{\rm inc*}] \tag{21}$$

is that due to the interference of the incident and the forward-scattered field. The latter term vanishes everywhere except along the straight line originating at the scattering object and extending in the incidence direction. Eq. (21) demonstrates again that it is the interference of the incident and forward-scattered fields that causes the extinction.

4. Coherency dyad of the total field

In order to characterize the directional flow and spatial distribution of the energy resulting from a scattering process, one must calculate the Poynting vector and the energy density of the total electromagnetic field. This, in turn, requires the knowledge of both the electric and the magnetic component of the field. It is attractive, however, to develop a simplified formalism that would involve the electric field only and would make feasible the solution of more involved problems such as the derivation of the radiative transfer equation [7]. Therefore, the aim of this section is to analyze whether the scattering process can be adequately described in terms of the so-called coherency dyad $\overleftrightarrow{\rho}(\mathbf{r}) = \mathbf{E}(\mathbf{r})\otimes\mathbf{E}^*(\mathbf{r})$, where $\otimes$ denotes the dyadic product of two vectors.

As before, we begin by representing the coherency dyad of the total field as the sum of three components:

$$\overleftrightarrow{\rho}(\mathbf{r}) = \overleftrightarrow{\rho}^{\rm inc} + \overleftrightarrow{\rho}^{\rm sca}(\mathbf{r}) + \overleftrightarrow{\rho}^{\rm int}(\mathbf{r}, \tag{22}$$

where

$$\overleftrightarrow{\rho}^{\rm inc} = \mathbf{E}^{\rm inc}(\mathbf{r})\otimes\mathbf{E}^{\rm inc*}(\mathbf{r}) = \mathbf{E}_0^{\rm inc}\otimes\mathbf{E}_0^{\rm inc*} \tag{23}$$

is the coherency dyad of the incident field,

$$\overleftrightarrow{\rho}^{\rm sca}(\mathbf{r}) = \mathbf{E}^{\rm sca}(\mathbf{r})\otimes\mathbf{E}^{\rm sca*}(\mathbf{r}) = \frac{1}{r^2}\mathbf{E}_1^{\rm sca}(\hat{\mathbf{r}})\otimes\mathbf{E}_1^{\rm sca*}(\hat{\mathbf{r}}) \tag{24}$$

is the coherency dyad of the scattered field, and the component

$$\overleftrightarrow{\rho}^{\rm int}(\mathbf{r}) = \mathbf{E}^{\rm inc}(\mathbf{r})\otimes\mathbf{E}^{\rm sca*}(\mathbf{r}) + \mathbf{E}^{\rm sca}(\mathbf{r})\otimes\mathbf{E}^{\rm inc*}(\mathbf{r})$$
$$= \frac{{\rm i}2\pi}{k_1 r^2}\{[\delta(\hat{\mathbf{n}}^{\rm inc}+\hat{\mathbf{r}})\exp(-{\rm i}2k_1 r) - \delta(\hat{\mathbf{n}}^{\rm inc}-\hat{\mathbf{r}})]\mathbf{E}_0^{\rm inc}\otimes\mathbf{E}_1^{\rm sca*}(\hat{\mathbf{r}})$$
$$+ [-\delta(\hat{\mathbf{n}}^{\rm inc}+\hat{\mathbf{r}})\exp({\rm i}2k_1 r) + \delta(\hat{\mathbf{n}}^{\rm inc}-\hat{\mathbf{r}})]\mathbf{E}_1^{\rm sca}(\hat{\mathbf{r}})\otimes\mathbf{E}_0^{\rm inc*}\} \tag{25}$$

can be interpreted as the result of interaction of the incident and scattered fields. The coherency dyad of the incident field directly yields the coherency and Stokes vectors of the incident field via

$$\mathbf{J}^{\rm inc} = \frac{1}{2}\sqrt{\frac{\varepsilon_1}{\mu_0}}\begin{bmatrix} \hat{\boldsymbol{\theta}}^{\rm inc}\cdot\overleftrightarrow{\rho}^{\rm inc}\cdot\hat{\boldsymbol{\theta}}^{\rm inc} \\ \hat{\boldsymbol{\theta}}^{\rm inc}\cdot\overleftrightarrow{\rho}^{\rm inc}\cdot\hat{\boldsymbol{\varphi}}^{\rm inc} \\ \hat{\boldsymbol{\varphi}}^{\rm inc}\cdot\overleftrightarrow{\rho}^{\rm inc}\cdot\hat{\boldsymbol{\theta}}^{\rm inc} \\ \hat{\boldsymbol{\varphi}}^{\rm inc}\cdot\overleftrightarrow{\rho}^{\rm inc}\cdot\hat{\boldsymbol{\varphi}}^{\rm inc} \end{bmatrix} \tag{26}$$

and

$$\mathbf{I}^{\rm inc} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & -1 & -1 & 0 \\ 0 & -{\rm i} & {\rm i} & 0 \end{bmatrix}\mathbf{J}^{\rm inc}, \tag{27}$$

respectively, where $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\varphi}}$ denote the unit vectors in the spherical coordinate system associated with the laboratory reference frame. Furthermore, we can rewrite Eq. (24) in the form

$$\overset{\leftrightarrow}{\rho}^{\text{sca}}(\mathbf{r}) = \frac{1}{r^2}[\overset{\leftrightarrow}{A}(\hat{\mathbf{r}}, \hat{\mathbf{n}}^{\text{inc}}) \cdot \mathbf{E}_0^{\text{inc}}] \otimes [\overset{\leftrightarrow}{A}(\hat{\mathbf{r}}, \hat{\mathbf{n}}^{\text{inc}}) \cdot \mathbf{E}_0^{\text{inc}}]^*, \tag{28}$$

where $\overset{\leftrightarrow}{A}(\hat{\mathbf{r}}, \hat{\mathbf{n}}^{\text{inc}})$ is the scattering dyadic defined by Eq. (2.26) of [6]. Recalling then that $\mathbf{E}_0^{\text{inc}} = \mathbf{E}_{0\theta}^{\text{inc}}\hat{\boldsymbol{\theta}}^{\text{inc}} + \mathbf{E}_{0\varphi}^{\text{inc}}\hat{\boldsymbol{\varphi}}^{\text{inc}}$ and $\mathbf{E}_1^{\text{sca}}(\hat{\mathbf{r}}) = E_{1\theta}^{\text{sca}}(\hat{\mathbf{r}})\hat{\boldsymbol{\theta}}^{\text{sca}} + E_{1\varphi}^{\text{sca}}(\hat{\mathbf{r}})\hat{\boldsymbol{\varphi}}^{\text{sca}}$ and using Eq. (2.30) of [6], we easily recover Eqs. (2.100) and (2.102) of [6], in which

$$\mathbf{J}^{\text{sca}}(r\hat{\mathbf{r}}) = \frac{1}{2}\sqrt{\frac{\varepsilon_1}{\mu_0}} \begin{bmatrix} \hat{\boldsymbol{\theta}}^{\text{sca}} \cdot \overset{\leftrightarrow}{\rho}^{\text{sca}}(r\hat{\mathbf{r}}) \cdot \hat{\boldsymbol{\theta}}^{\text{sca}} \\ \hat{\boldsymbol{\theta}}^{\text{sca}} \cdot \overset{\leftrightarrow}{\rho}^{\text{sca}}(r\hat{\mathbf{r}}) \cdot \hat{\boldsymbol{\varphi}}^{\text{sca}} \\ \hat{\boldsymbol{\varphi}}^{\text{sca}} \cdot \overset{\leftrightarrow}{\rho}^{\text{sca}}(r\hat{\mathbf{r}}) \cdot \hat{\boldsymbol{\theta}}^{\text{sca}} \\ \hat{\boldsymbol{\varphi}}^{\text{sca}} \cdot \overset{\leftrightarrow}{\rho}^{\text{sca}}(r\hat{\mathbf{r}}) \cdot \hat{\boldsymbol{\varphi}}^{\text{sca}} \end{bmatrix} \tag{29}$$

and

$$\mathbf{I}^{\text{sca}}(r\hat{\mathbf{r}}) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & -1 & -1 & 0 \\ 0 & -\mathrm{i} & \mathrm{i} & 0 \end{bmatrix} \mathbf{J}^{\text{sca}}(r\hat{\mathbf{r}}). \tag{30}$$

Finally, by integrating the coherency dyad of the total field over the surface of detector 1 in Fig. 1 and applying similar algebra, we recover Eqs. (2.136) and (2.138) of [6].

Thus, the use of the coherency dyad to describe the electromagnetic scattering process in the far-field zone appears to be consistent with the main results of Sections 2.6 and 2.7 of [6]. However, one encounters a problem when the response of the detector facing the exact backscattering direction is being considered. When the right-hand side of Eq. (25) is integrated over the surface of this detector, the term proportional to $\delta(\hat{\mathbf{n}}^{\text{inc}} + \hat{r})$ gives a non-zero contribution due to apparent interference of the incident and the backscattered field. However, in view of the discussion in Sections 2 and 3, this contribution is unphysical. Indeed, the effect of interference of the incident and backscattered fields is annihilated by the real filter Re on the right-hand side of Eq. (9) and by the fact that the corresponding electric and magnetic contributions to $\langle U^{\text{ext}}(\mathbf{r}, t)\rangle_t$ in Eq. (21) cancel each other. It is thus clear that one must exercise caution if the coherency dyad is used as a basic characteristic of the electromagnetic scattering process.

5. Conclusions

In this paper, we have supplemented previous analyses of the phenomenon of extinction by considering the spatial distribution of the electromagnetic energy density of the total field as well as its Poynting vector. We have demonstrated again that the extinction is the result of interference of the incident and the forward-scattered field and that the full electromagnetic treatment of the scattering process has natural built-in filters that zero out the contribution of the interference of the incident and the back-scattered field.

Such built-in filters may be absent in theoretical formalisms based on quantities other than the Poynting vector and the electromagnetic energy density. Therefore, one must carefully establish the relationship between the specific quantities used to characterize the scattering process theoretically and the basic physical observables in order to avoid false interpretation of the theoretical results in terms of spurious physical effects.

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